Received: *March 22, 1999,* Accepted: *May 27, 1999*

Neutrino masses and mixing with non-anomalous abelian flavor symmetries

Yosef Nir and Yael Shadmi

Department of Particle Physics
Weizmann Institute of Science, Rehovot 76100, Israel
E-mail: ftnir@clever.weizmann.ac.il, yshadmi@wicc.weizmann.ac.il

Abstract: The experimental data on atmospheric and solar neutrinos are used to test the framework of non-anomalous Abelian horizontal gauge symmetries with only three light active neutrinos. We assume that the hierarchy in mass-squared splittings is not accidental and that the small breaking parameters are not considerably larger than 0.2. We find that the small angle MSW solution of the solar neutrino problem can only be accommodated if the $\nu_\mu - \nu_\tau$ mass hierarchy depends on the charges of at least three sterile neutrinos. The large angle MSW solution can be accommodated in simpler models if ν_e and ν_μ form a pseudo-Dirac neutrino, but it is difficult to induce large enough deviation from maximal mixing. The vacuum oscillation solution can be accommodated rather simply. We conclude that it is possible to accommodate the neutrino parameters in the framework of Abelian horizontal symmetries, but it seems that these parameters by themselves will not provide convincing evidence for this framework.

Keywords: Solar and Atmospheric Neutrinos, Neutrino Physics, Supersymmetric Standard Model.

JHEP05(1999)023

Contents

1. Introduction and results

Approximate Abelian horizontal symmetries can explain the smallness and the hierarchy in the flavor parameters – fermion masses and mixing angles – in a natural and simple way [1]. One can think of three types of evidence for such symmetries: First, the full theory involves fields that are related to the spontaneous symmetry breaking and to the communication of the breaking to the observable sector. Direct discovery of such particles is, however, very unlikely because constraints from flavor changing neutral current (FCNC) processes and from Landau poles imply that they should be very heavy [2]. Second, the supersymmetric flavor parameters are also determined by the selection rules of the horizontal symmetry [3, 4]. (This is likely to be the case if supersymmetry breaking is mediated to the observable sector by Planck-scale interactions [5]; in contrast, gauge mediation would erase the effects of the horizontal symmetry from the sfermion flavor parameters.) The spectrum of supersymmetric particles and, in particular, supersymmetric effects on FCNC and on CP violation could then provide evidence for the horizontal symmetry. Third, it could be that the Yukawa parameters themselves obey simple order of magnitude relations that follow from the horizontal symmetry [6]. In this context, Abelian horizontal symmetries have much more predictive power in the lepton sector than in the quark sector [7]. Neutrino parameters provide then an important input for testing and refining this framework.

As concerns neutrino parameters, recent measurements of the flux of atmospheric neutrinos (AN) suggest the following mass-squared difference and mixing between ν_μ and ν_τ [8]:

$$\Delta m^2_{23} \sim 2 \times 10^{-3}\ eV^2\,, \qquad \sin^2 2\theta_{23} \sim 1\,. \tag{1.1}$$

On the other hand, measuremnts of the solar neutrino (SN) flux can be explained by one of the following three options for the parameters of $\nu_e - \nu_x$ ($x = \mu$ or τ) oscillations (for a recent analysis, see [9]):

	$\Delta m^2_{1x}\ [eV^2]$	$\sin^2 2\theta_{1x}$
MSW(SMA)	5×10^{-6}	6×10^{-3}
MSW(LMA)	2×10^{-5}	0.8
VO	8×10^{-11}	0.8

(1.2)

Here MSW refers to matter-enhanced oscillations, VO refers to vacuum oscillations, and SMA (LMA) stand for small (large) mixing angle. Only central values are quoted for the various parameters.

Our basic assumption will be that eqs. (1.1) and (1.2) imply that the ratio between the mass splittings is suppressed by the small breaking parameter of an Abelian horizontal symmetry, $\lambda \sim 0.2$, while the $\nu_\mu - \nu_\tau$ mixing angle is not:

$$\begin{aligned} &\sin\theta_{23} \sim 1\,,\\ &\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \sim \begin{cases} \lambda^2 - \lambda^4 & MSW,\\ \lambda^{10} - \lambda^{12} & VO. \end{cases} \end{aligned}$$

We note, however, that it is not impossible that, if the solar neutrino problem is a result of the MSW mechanism, the ratio between $\Delta m^2_{\rm SN}$ and $\Delta m^2_{\rm AN}$ is accidentally, rather than parametrically, suppressed [10, 11]. Then, the analysis of this work and of ref. [12] is irrelevant, and the framework of Abelian horizontal symmetries can accommodate the neutrino parameters in a simple way.

In a previous work [12] we investigated the implications of (1.3) for models where an Abelian horizontal symmetry H is broken by a *single* small parameter. (For recent related work, see [13, 14].) This assumption is best-motivated in models where the horizontal symmetry is an anomalous $\mathrm{U}(1)_H$ gauge symmetry [15, 16]. The anomaly is cancelled by the Green-Schwarz mechanism [17]. The contribution of the Fayet-Iliopoulos term to the D-term cancels against the contribution from a VEV of a Standard Model (SM) singlet field S with H-charge that is opposite in sign to tr$H$. (Without loss of generality, we choose the H-charge of S to be -1). The information about the breaking is communicated to the observable sector (MSSM)

JHEP05(1999)023

at the string scale. The ratio $\lambda = \langle S \rangle / m_{\rm Pl} \sim {\rm tr}H/192\pi^2$ [18, 19] provides the small breaking parameter of H. The single VEV assumption is also plausible if the horizontal symmetry is discrete. In the single VEV framework, as explained in [12], it is non-trivial to get large mixing together with a large hierarchy as implied by 1.3. To obtain that one needs to invoke either holomorphic zeros or discrete symmetries, often with a symmetry group that is a direct product of two factors.

The situation is different if the Abelian horizontal symmetry is a non-anomalous gauge symmetry. If supersymmetry is not to be broken at the scale of spontaneous H-breaking, then H should be broken along a D-flat direction. The simplest possibility then is that two scalars, S and $\bar{S}$, of opposite H-charges (say, ± 1) assume equal VEVs, $\langle S \rangle = \langle \bar{S} \rangle$ [20, 21, 11]. In this work we investigate the implications of (1.3) in the two-VEVs framework.

It is straightforward to see that our previous mechanisms are irrelevant in the new framework. First, with discrete symmetries there is no motivation for the two-VEVs scenario. There is also no sense in talking about negative charges. Second, with VEVs of opposite charges there can be no holomorphic zeros. On the other hand, this framework is in some sense less predictive and, consequently, allows new mechanisms to accommodate simultaneous large mixing and mass hierarchy between neutrinos. In particular, this situation can be obtained naturally with a single U(1) horizontal symmetry.

As we shall see, if the small-angle MSW solution is found to be valid, it would mean that the light neutrino masses depend crucially on the horizontal charges of (at least three) heavy sterile neutrinos.

If, on the other hand, the large-angle MSW solution is confirmed, two possibilities are allowed. One is that the light neutrino masses are again affected by the charges of heavy sterile neutrinos. The second is that the MSW solution corresponds to oscillations between the first and second generation neutrinos, which form a pseudo-Dirac neutrino. In the latter case, there is another constraint on the neutrino parameters, $\sin 2\theta_{12} < 0.9$. This constraint is not simple to satisfy since the mixing in the pseudo-Dirac case is close to maximal [22]. In fact, the deviation from maximal mixing is at most $\mathcal{O}(\lambda^2)$. These models are therefore marginally viable. They are only consistent if the $\mathcal{O}(\lambda^2)$ correction is enhanced by about three.

The plan of this paper is as follows. In section 2 we specify our theoretical framework. In section 3 we argue that, in the framework of Abelian horizontal symmetries, ν_μ and ν_τ cannot pair to a pseudo-Dirac neutrino. In section 4 we review previous results in the framework of an anomalous U(1) symmetry. In section 5 we analyze in detail the lepton parameters in the framework of non-anomalous Abelian gauge symmetry. (Proofs of some of the statements of this section are given in the appendix.) We conclude in section 6, emphasizing the difficulties that the data on solar and atmospheric neutrinos pose to the framework of Abelian horizontal gauge symmetries.

JHEP05(1999)023

2. The theoretical framework

Our theoretical framework is defined as follows. We consider a low energy effective theory with particle content that is the same as in the Supersymmetric Standard Model. In addition to supersymmetry and to the Standard Model gauge symmetry, there is an approximate $U(1)_H$ symmetry that is broken by two small parameters λ and $\bar{\lambda}$ [23, 20, 21, 11]. The two parameters are assumed to be equal in magnitude:

$$\lambda = \bar{\lambda} \sim 0.2\,. \tag{2.1}$$

(The choice of numerical value comes from the quark sector, where the largest small parameter is $\sin\theta_C = 0.22$.) To derive selection rules, we attribute to the breaking parameters $U(1)_H$ charges:

$$H(\lambda) = +1\,, \qquad H(\bar{\lambda}) = -1\,. \tag{2.2}$$

Then, the following selection rule applies: *Terms in the superpotential or in the Kahler potential that carry (integer) H-charge n are suppressed by $\lambda^{|n|}$.*[1]

We assume that the active neutrinos (coming from lepton doublet supermultiplets L_i), are light because of a seesaw mechanism involving heavy sterile neutrinos (coming from singlet supermultiplets N_i). However, the relations (1.3) do not necessarily depend on the charges of all active and sterile neutrinos. For example, if ν_e is much lighter than all other neutrinos, it does not enter (1.3). Likewise, if all sterile neutrinos may be integrated out at the H-breaking scale, they do not affect the effective light neutrino mass matrix. We will refer to a model containing n_a relevant active neutrinos and n_s relevant sterile neutrinos as an (n_a, n_s) model.

We will only consider models where all fields carry integer H-charges (in units of the charge of the breaking parameters). It is possible that some or all of the lepton fields carry half-integer charges [22]. Then there is a residual, unbroken discrete symmetry. Such models can be phenomenologically viable and lead to interesting predictions. We leave the investigation of this class of models to a future publication [24].

Note that if the horizontal symmetry is a continuous symmetry with a single breaking parameter, as would be the case for an anomalous U(1), the selection rule stated above is modified. Superpotential terms that carry negative H-charge cannot appear, as they would require powers of $\lambda^\dagger$, which is forbidden by holomorphy [2]. We refer to these absent terms as "holomorphic zeros".

The theory is limited in the sense that it cannot predict the exact coefficients of $\mathcal{O}(1)$ for the various terms. Wherever we use the symbol "$\sim$" below we mean to say that the unknown coefficients of $\mathcal{O}(1)$ are omitted.

[1] The same selection rule would apply in a theory with a single breaking parameter and no supersymmetry.

RGE effects could enhance the neutrino mixing angle [25, 26, 11]. The enhancement can take place if $\tan\beta$ is large and if the mass ratio between the corresponding neutrinos is not small. These enhancement effects are not important in our framework and we will not take them into account.

3. On pseudo-Dirac neutrinos

As a first step in our discussion, we would like to make a general comment about pseudo-Dirac neutrinos in the framework of Abelian horizontal symmetries. In many of our examples (here and in [12]), two of the three active neutrinos pair to form a pseudo-Dirac neutrino. In all of these examples, the parameters of the pseudo-Dirac neutrino (maximal mixing and very small mass splitting) are fitted to solve the SN problem. Since AN observations seem to favor *maximal*, and not just generic $\mathcal{O}(1)$, mixing, one may wonder whether we could find a model where, indeed, the mass splitting between the components of the pseudo-Dirac neutrino corresponds to $\Delta m^2_{\rm AN}$. We argue now that this is impossible.

The argument goes as follows. Let us define $m_{\rm pD}$ to be the mass of the pseudo-Dirac neutrino and $\delta_{\rm pD} \ll m_{\rm pD}$ to be the mass splitting between its components. An Abelian symmetry cannot give an exact relation between three entries in the mass matrix. (The symmetric structure of M_ν relates pairs of entries, which enables us to find models with a pseudo-Dirac neutrino.) Therefore, the mass-squared splitting between the pseudo-Dirac neutrino and the other mass eigenstate is at least $\mathcal{O}(m^2_{\rm pD})$.[2] On the other hand, the mass-squared splitting between the components of the pseudo-Dirac neutrino is $\mathcal{O}(\delta_{\rm pD} m_{\rm pD})$. Since $\delta_{\rm pD} \ll m_{\rm pD}$, the mass-squared splitting between the components of the pseudo-Dirac neutrino is much smaller than the mass-squared splitting between the pseudo-Dirac neutrino and the third mass eigenstate. Therefore, the former corresponds to $\Delta m^2_{\rm SN}$ and the latter to $\Delta m^2_{\rm AN}$.[3]

It is worth emphasizing that the discussion of this section applies to any Abelian symmetry, continuous or discrete.

4. Continuous symmetry with a single breaking parameter

Before moving on to our discussion of continuous symmetries with two breaking parameters, let us recall some results of [12] in the single VEV framework.

The main obstacle in obtaining, within the single-VEV framework, large mixing between hierarchically separated neutrinos, can be explained as follows. Consider a single $\mathrm{U}(1)_H$ symmetry. Large mixing between, say, ν_2 and ν_3 can only be obtained in two cases: either the H-charges of the lepton doublets are equal [7], $H(L_2) = H(L_3)$,

[2]It is of course possible to fine tune the $\mathcal{O}(1)$ coefficients to get a stronger degeneracy [27].

[3]If one considers *only* the AN problem, then it is of course possible that ν_μ and ν_τ form a pesudo-Dirac neutrino [28].

or they are opposite [13], $H(L_2) = -H(L_3)$. In the first case the mixing is $\mathcal{O}(1)$ but the masses are of the same order of magnitude, $m(\nu_2) \sim m(\nu_3)$. In the second case, to a good approximation, the mixing is maximal, $\sin^2 2\theta_{23} = 1$ and the masses are equal, $m(\nu_2) = m(\nu_3)$. (This is the case of a pseudo-Dirac neutrino.) In either case, there is no mass hierarchy. If the symmetry is continuous but more complicated, say $\mathrm{U}(1)_1 \times \mathrm{U}(1)_2$ with the respective breaking parameters of order λ^m and λ^n, then one can still define an *effective H-charge*,

$$H_{\text{eff}} = mH_1 + nH_2 \,. \tag{4.1}$$

Large mixing can only be obtained for $H_{\text{eff}}(L_2) = \pm H_{\text{eff}}(L_3)$, so that, again, there is no mass hierarchy. This conclusion can only be evaded if holomorphic zeros appear in the neutrino mass matrix such that one of the two mass eigenvalues vanishes. In other words, in the single VEV framework, when considering ν_2 and ν_3 only, the only way to get large mixing and a large hierarchy between them is to make ν_2 massless. Clearly though, we need to generate a mass for ν_2 as well in order to account for 1.3. This can be arranged by having ν_2 combine with ν_1 to form a pseudo-Dirac neutrino. But then $\sin 2\theta_{12}$ is large, and we cannot obtain the small angle MSW solution.

In the two VEV framework, the notion of effective charge is not very useful. It is the absence of similarly powerful constraints that, on the one hand, makes the two VEV models less predictive but, on the other hand, allows one to accommodate the neutrino parameters more easily.

5. Continuous symmetry with two breaking parameters

5.1 No pseudo-Dirac neutrino

We start our investigation of Abelian horizontal symmetries with two breaking parameters by assuming that the solution of the SN problem does not involve a pseudo-Dirac neutrino. Then the hierarchy of mass-squared splittings is simply related to the hierarchy of masses:

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \sim \frac{m_2^2}{m_3^2} \,. \tag{5.1}$$

The case $m_2 \ll m_1$ is only relevant to the VO solution of the SN problem and does not affect our conclusions in this subsection. These assumptions allow us to investigate the relevant parameters in a $(2, n_s)$ framework.

A simple mechanism for inducing large mixing between hierarchically separated masses can be demonstrated in a simple $(2,0)$ model. In this model, the lepton mass matrices have the form:

$$M_\nu \sim \frac{\langle\phi_u\rangle^2}{M} \begin{pmatrix} \lambda^{2|H(L_2)|} & \lambda^{|H(L_2)+H(L_3)|} \\ \lambda^{|H(L_2)+H(L_3)|} & \lambda^{2|H(L_3)|} \end{pmatrix}, \tag{5.2}$$

$$M_{\ell^\pm} \sim \langle\phi_d\rangle \begin{pmatrix} \lambda^{|H(L_2)+H(\bar\ell_2)|} & \lambda^{|H(L_2)+H(\bar\ell_3)|} \\ \lambda^{|H(L_3)+H(\bar\ell_2)|} & \lambda^{|H(L_3)+H(\bar\ell_3)|} \end{pmatrix}. \tag{5.3}$$

To have $m_2 \ll m_3$, we need $|H(L_2)| \neq |H(L_3)|$. We can still get a mixing of $\mathcal{O}(1)$ if

$$H(L_2) + H(L_3) = -2H(\bar{\ell}_3)\,. \tag{5.4}$$

In a general (n_a, n_s) model, large mixing could arise also from the neutrino Dirac mass matrix or the Majorana mass matrix of the sterile neutrinos. But in all these cases, it is required, similarly to (5.4), that

$$H(L_2) - H(L_3) = 0 \pmod 2\,. \tag{5.5}$$

Eq. (5.5) has interesting implications for the mass hierarchy. From eq. (5.2) we learn that

$$\frac{m_2^2}{m_3^2} \sim \lambda^{4[|H(L_2)+H(L_3)|-2|H(L_3)|]} \sim \lambda^{8n} \qquad (n = \text{ integer})\,. \tag{5.6}$$

This creates a phenomenological problem. If the hierarchy is λ^0, $\Delta m^2_{\rm SN}$ is too large. The next weakest possibilities are λ^8 or λ^{16}. But for the MSW solutions we need λ^{2-4} and for the VO solution we need λ^{10-12}. We can achieve neither in this framework. The MSW solutions are particularly disfavored; the VO solution may still correspond to the λ^8 hierarchy if λ is actually close to 0.1 (rather than the value of 0.2 that we usually use).[4]

Of course, we have only considered the (2,0) framework. However, one can show that eq. (5.6) holds also in the (2,2) case. Within $(2, n_s \geq 3)$ models, however, the hierarchy is an integer power of λ^4 and, consequently, could be milder.[5] A proof of these statements can be found in the Appendix. We conclude then that one of the following options has to happen in order to achieve neutrino parameters that are consistent with the MSW solutions:

(i) There are at least three sterile neutrinos that affect the mass hierarchy m_2/m_3. (A $(2,3)$ model which accommodates the MSW(SMA) solution can be found in [30].)

(ii) ν_e and ν_μ form a pesudo-Dirac neutrino. This case is, of course, relevant only to the MSW(LMA) solution.

A simple example of how the VO solution can be implemented in this framework, with $\lambda \sim 0.1$, is achieved with the following set of charges:

$$\begin{array}{lll} L_1(+9)\,, & L_2(+3)\,, & L_3(+1)\,, \\ \ell_1(-15)\,, & \ell_2(-6)\,, & \bar{\ell}_3(-2)\,. \end{array} \tag{5.7}$$

[4] The MSW solution can still be accommodated if the small breaking parameters are large, $\lambda \sim 0.5$ [29].

[5] We thank F. Feruglio for pointing out a $(2,3)$ example with λ^4 hierarchy.

JHEP05(1999)023

The neutrino mass matrix is

$$M_\nu \sim \frac{\langle\phi_u\rangle^2}{M}\begin{pmatrix}\lambda^{18} & \lambda^{12} & \lambda^{10}\\ \lambda^{12} & \lambda^{6} & \lambda^{4}\\ \lambda^{10} & \lambda^{4} & \lambda^{2}\end{pmatrix}, \tag{5.8}$$

which yields

$$\frac{\Delta m^2_{\rm SN}}{\Delta m^2_{\rm AN}} \sim \lambda^8 \,. \tag{5.9}$$

For the charged lepton mass matrix we find

$$M_\ell \sim \langle\phi_d\rangle\begin{pmatrix}\lambda^{6} & \lambda^{3} & \lambda^{7}\\ \lambda^{12} & \lambda^{3} & \lambda\\ \lambda^{14} & \lambda^{5} & \lambda\end{pmatrix}, \tag{5.10}$$

which gives $\sin\theta_{23} \sim 1$ and, for $\tan\beta \sim \lambda^{-2}$, the required charged lepton mass hierarchy.

5.2 A pseudo-Dirac neutrino: hierarchy of mass splittings without hierarchy of masses

A large $\nu_\mu{-}\nu_\tau$ mixing, relevant to the AN problem, and a large $\nu_e{-}\nu_\mu$ mixing, relevant to the SN problem, could arise from very different mechanisms: $\mathcal{O}(1)$ $\nu_\mu - \nu_\tau$ mixing from unequal charges (as discussed in the previous subsection), and maximal $\nu_e - \nu_\mu$ mixing from their pairing to a pseudo-Dirac combination. Such a situation opens up the interesting possibility that there is actually no mass hierarchy: All three neutrino masses may be of the same order of magnitude, which is the scale set by AN, with the mass-squared *splittings* hierarchically separated.

It is simple to see that all three neutrino masses are of the same order of magnitude if we take

$$|H(L_1)+H(L_2)| = 2|H(L_3)|\,. \tag{5.11}$$

Instead of (5.1) we now have

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \sim \frac{m_{\rm pD}\delta_{\rm pD}}{m_3^2} \sim \frac{\delta_{\rm pD}}{m_{\rm pD}}\,. \tag{5.12}$$

The dependence of this ratio on the lepton charges is given by

$$\frac{\Delta m^2_{\rm SN}}{\Delta m^2_{\rm AN}} \sim \lambda^{2[|H(L_2)|-|H(L_3)|]} \lesssim \lambda^4\,. \tag{5.13}$$

In contrast to (5.6), the mass hierarchy is appropriate for the MSW parameters. Yet, the MSW(LMA) solution cannot be achieved because the deviation from maximal mixing is suppressed by at least $\mathcal{O}(\lambda^4)$. The reason is that a deviation of $\mathcal{O}(\lambda^2)$ can be achieved only if $H(L_1)-H(L_2)$ is odd, but this is impossible because of (5.11).

Therefore, in this class of models, only the VO solution can be accommodated. We now demonstrate this by an explicit example.

Consider the following set of H-charges for the lepton fields in the (3,0) framework:

$$\begin{array}{lll} L_1(+7)\,, & L_2(-5)\,, & L_3(+1)\,, \\ \bar{\ell}_1(-15)\,, & \bar{\ell}_2(+10)\,, & \bar{\ell}_3(+2)\,. \end{array} \tag{5.14}$$

The neutrino mass matrix is of the form

$$M_\nu \sim \frac{\langle \phi_u \rangle^2}{M} \begin{pmatrix} \lambda^{14} & A\lambda^2 & \lambda^8 \\ A\lambda^2 & \lambda^{10} & \lambda^4 \\ \lambda^8 & \lambda^4 & B\lambda^2 \end{pmatrix}. \tag{5.15}$$

For later purposes we explicitly wrote down the $\mathcal{O}(1)$ coefficients, A and B, of the dominant entries. We see that all three neutrinos have masses of the same order of magnitude, that is

$$m(\nu_i) \sim \frac{\langle \phi_u \rangle^2}{M}\lambda^2\,, \qquad \text{for } i = 1,2,3\,. \tag{5.16}$$

The mass splittings are, however, hierarchical:

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \sim \lambda^8\,, \tag{5.17}$$

which fits the VO solution for $\lambda \sim 0.1$. A large $2-3$ mixing is obtained from the charged lepton sector:

$$M_\ell \sim \langle \phi_d \rangle \begin{pmatrix} \lambda^8 & \lambda^{17} & \lambda^9 \\ \lambda^{20} & \lambda^5 & \lambda^3 \\ \lambda^{14} & \lambda^{11} & \lambda^3 \end{pmatrix}. \tag{5.18}$$

We learn that

$$\sin^2 2\theta_{12} \simeq 1\,, \qquad \sin\theta_{13} \sim \lambda^6\,, \qquad \sin\theta_{23} \sim 1\,. \tag{5.19}$$

Note that this scenario of hierarchical mass-squared splittings is not included in ref. [22]. The form of the neutrino mass matrix (5.15) in the charged lepton mass basis is given to a good approximation by

$$M_\nu \sim \frac{\langle \phi_u \rangle^2}{M}\lambda^2 \begin{pmatrix} 0 & Ac & As \\ Ac & Bs^2 & Bcs \\ As & Bcs & Bc^2 \end{pmatrix}, \tag{5.20}$$

where $c \equiv \cos\theta_{23}$ and $s \equiv \sin\theta_{23}$. Indeed, this corresponds to neither of the two forms advocated in [22]. It is amusing to note, however, that it can be presented as the sum of these two forms.

JHEP05(1999)023

5.3 Large mixing from equal effective charges

A different way of obtaining large mixing together with a large hierarchy is the analog of the 'holomorphic zeros' mechanism of ref. [12]. In both frameworks we take the horizontal symmetry to be $U(1)_1 \times U(1)_2$, with equal effective charges (see eq. 4.1) for L_2 and L_3, so that the $2-3$ mixing is $\mathcal{O}(1)$. The separate charges can, however, be chosen so as to induce a holomorphic zero in the single VEV framework and to suppress one of the masses in the two VEV framework.

As an example consistent with the MSW(LMA) solution, consider the following set of charges for the lepton fields within a (3,0) model [12]:

$$\begin{array}{ccc} L_1(1,0)\,, & L_2(-1,1)\,, & L_3(0,0)\,, \\ \bar\ell_1(3,4)\,, & \bar\ell_2(3,2)\,, & \bar\ell_3(3,0)\,. \end{array} \tag{5.21}$$

The lepton mass matrices are of the form

$$M_\nu \sim \frac{\langle\phi_u\rangle^2}{M}\begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & \lambda^4 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}, \qquad M_{\ell^\pm} \sim \langle\phi_d\rangle \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^7 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^5 & \lambda^3 \end{pmatrix}. \tag{5.22}$$

Without the positively charged $\bar\lambda_1$, the (2,2), (2,3) and (3,2) entries in M_ν would vanish because of holomorphy [12]. Here, the holomorphic zeros are lifted, but the new entries are small and affect neither the mass hierarchy nor the mixing. Thus, the analysis of [12] is still valid, yielding

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \sim \lambda^3\,, \qquad \sin 2\theta_{12} = 1 - \mathcal{O}(\lambda^2)\,, \qquad s_{23} \sim 1\,, \qquad s_{13} \sim \lambda\,. \tag{5.23}$$

The VO solution is similarly obtained with

$$\begin{array}{ccc} L_1(1,-4)\,, & L_2(-2,2)\,, & L_3(0,0)\,, \\ \bar\ell_1(6,5)\,, & \bar\ell_2(3,2)\,, & \bar\ell_3(3,0)\,. \end{array} \tag{5.24}$$

This gives

$$M_\nu = \begin{pmatrix} \lambda^{10} & \lambda^3 & \lambda^5 \\ \lambda^3 & \lambda^8 & \lambda^4 \\ \lambda^5 & \lambda^4 & 1 \end{pmatrix}, \tag{5.25}$$

yielding $\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \sim \lambda^{11}$.

6. Conclusions

In models with a non-anomalous Abelian horizontal gauge symmetry, one expects that the symmetry is spontaneously broken by fields of opposite horizontal charges that acquire equal VEVs. We find two general mechanisms by which such models can accommodate the neutrino parameters that explain both the atmospheric neutrino and the solar neutrino problems. In particular, these mechanisms allow for

$$\sin\theta_{23} \sim 1\,, \qquad \frac{\Delta m^2_{12}}{\Delta m^2_{23}} \ll 1\,. \tag{6.1}$$

The possible scenarios divide into two general classes:

(i) The three neutrino masses are hierarchical. Then the hierarchy between the mass-squared splittings of eq. (6.1) is an integer power of λ^4. The bound can only be saturated in models where at least three sterile neutrinos affect the mass hierarchy. Otherwise, the hierarchy is an integer power of λ^8 which is inconsistent with the MSW solutions. The VO solution can be accommodated in the simple models that have λ^{8n} hierarchy if the small breaking parameter is somewhat smaller than the 'canonical' value of 0.2 related to the Cabibbo mixing.

(ii) The two lighter neutrinos form a pseudo-Dirac neutrino. The mixing related to the solar neutrino solution is then close to maximal, so that obviously only large angle solutions to the SN problem are possible. It is not simple, though not impossible, to obtain large enough deviation from maximal mixing as necessary for the large angle MSW solution. The VO solution is, again, simply accommodated. A special case in this class is that of three same order-of-magnitude neutrino masses, with hierarchical splittings.

We emphasize that our arguments are not valid for *discrete* Abelian symmetries [12]. Moreover, they can be circumvented even if the symmetry is continuous but the spontaneous symmetry breaking is not complete, leaving a residual exact discrete symmetry [22, 24]. In particular, it has been demonstrated that the MSW(SMA) solution can be easily generated in these cases [12, 22].

While the conclusion of both this work and of ref. [12] is that large mixing and large hierarchy can be accommodated in the framework of Abelian horizontal symmetries, we would still like to emphasize the following points:

a. The most predictive class of models of Abelian horizontal symmetries is that of an anomalous U(1) with holomorphic zeros having no effect on the physical parameters [7]. The various solutions suggested here and in [12] require that either the symmetry is non-anomalous with two breaking parameters, or the symmetry is discrete, or that holomorphic zeros do play a role. In all these cases there is a loss of predictive power. If, indeed, (6.1) holds in nature, it would mean that neutrino parameters by themselves will not make a convincing case for the Abelian horizontal symmetry idea, even if they cannot rule it out.

JHEP05(1999)023

b. We argued here that, if (6.1) holds, the neutrino parameters that correspond to the atmospheric neutrino oscillations are not related to a pseudo-Dirac neutrino. Consequently, while Abelian horizontal symmetries allow for $\mathcal{O}(1)$ $\nu_\mu - \nu_\tau$ mixing, they cannot explain *maximal* mixing (except as an accidental result). If the case for $\sin^2 2\theta_{23} = 1$ is experimentally made with high accuracy, and the solar neutrino problem is indeed solved by neutrino oscillations, the framework of Abelian horizontal symmetries would become less attractive.

Acknowledgments

We are grateful to Ferruccio Feruglio for providing us with a counter-example to statements made in the original version of this paper. We thank Enrico Nardi for helpful discussions. This work was supported in part by the United States – Israel Binational Science Foundation (BSF) and by the Minerva Foundation (Munich).

JHEP05(1999)023

A. Mass hierarchy with large mixing

We study models with n_a active neutrinos and n_s sterile ones ('(n_a, n_s) models'). We will argue that, as concerns the implications for Δm^2_{12}, the models divide into three classes:

a. Effective $(2, n_s \geq 3)$ models, where we find that $\Delta m^2_{12}/\Delta m^2_{23} \sim \lambda^{4n}$ ($n =$ integer).

b. Effective $(2, 2)$ models, where we find that $\Delta m^2_{12}/\Delta m^2_{23} \sim \lambda^{8n}$ ($n =$ integer).

c. Models where ν_e and ν_μ form a pseudo-Dirac neutrino, where $\Delta m^2_{12} \ll m^2_{1,2}$.

The first step in our argument is to show that the large mixing between ν_2 and ν_3 requires that the horizontal charges of L_2 and L_3 obey

$$H(L_2) - H(L_3) = 0 \pmod 2\,. \tag{A.1}$$

There are three possible sources (in the interaction basis) for large mixing: (i) the charged lepton mass matrix M_ℓ, (ii) the neutrino Dirac mass matrix $M_\nu^{\rm Dir}$, and (iii) the sterile neutrino Majorana mass matrix $M_{\nu_s}^{\rm Maj}$. We now examine the three mechanisms in turn.

(i) If the large mixing arises from M_ℓ, then

$$(M_\ell)_{23} \sim (M_\ell)_{33} \Longrightarrow |H(L_2) + H(\bar\ell_3)| = |H(L_3) + H(\bar\ell_3)|\,. \tag{A.2}$$

Therefore, either $H(L_2) = H(L_3)$, or $H(L_2) + H(L_3) = -2H(\bar\ell_3)$. In either case (A.1) holds.

(ii) If the large mixing arises from $M_\nu^{\rm Dir}$, then a similar argument holds with $H(\bar\ell_3)$ replaced by $H(N_3)$.

(iii) Let us examine the conditions for large mixing induced by $M_{\nu_s}^{\rm Maj}$. To do so, we consider a (2,2) model. We define the relevant matrix elements by

$$(M_{\nu_s}^{\rm Maj})^{-1} = \begin{pmatrix} r_{22} & r_{23} \\ r_{23} & r_{33} \end{pmatrix}. \tag{A.3}$$

For simplicity, we take $M_\nu^{\rm Dir}$ to be diagonal (consistent with our assumption that the large mixing does not come from this matrix):

$$M_\nu^{\rm Dir} = \begin{pmatrix} d_2 & \\ & d_3 \end{pmatrix}. \tag{A.4}$$

Then

$$M_{\nu_a}^{\rm Maj} = (M_\nu^{\rm Dir})(M_{\nu_s}^{\rm Maj})^{-1}(M_\nu^{\rm Dir})^T = \begin{pmatrix} d_2^2 r_{22} & d_2 d_3 r_{23} \\ d_2 d_3 r_{23} & d_3^2 r_{33} \end{pmatrix}. \tag{A.5}$$

Large mixing can be induced in two cases: first, $d_2 d_3 r_{23} \gg d_3^2 r_{33}$ which leads to a pesudo-Dirac neutrino in contrast to our assumptions; second, $d_2 d_3 r_{23} \sim d_3^2 r_{33}$, which can be achieved with

$$\frac{d_2 r_{23}}{d_3 r_{33}} \sim \lambda^{|H(L_2)+H(N_2)|-|H(L_3)+H(N_3)|+|H(N_2)+H(N_3)|-2|H(N_2)|} \sim 1\,. \tag{A.6}$$

The condition on the exponent is then of the form

$$a_2 H(L_2) + a_3 H(L_3) + 2b_2 H(N_2) + 2b_3 H(N_3) = 0\,, \tag{A.7}$$

where $a_2, a_3 = \pm 1$, $b_3 = 0, \pm 1$ and $b_2 = 0, \pm 1, \pm 2$. Clearly, it leads to (A.1).

The second step is to find the hierarchy between the masses in terms of the lepton charges. If no pair among the active neutrinos forms a pseudo-Dirac state, then we can estimate the mass ratio from

$$\frac{m_2}{m_3} \sim \frac{\det M^{(n_s+2)} \det M^{(n_s)}}{[\det M^{(n_s+1)}]^2}\,. \tag{A.8}$$

Here, $\det M^{(n_s)}$ is the product of the masses of the sterile neutrinos, which is approximately equal to $\det M_{\nu_s}^{\rm Maj}$, and $\det M^{(n_s+n'_a)}$ is the product of the masses of the sterile neutrinos and the masses of the n'_a heaviest active neutrinos. To estimate the masses, we use

$$(M_\nu^{\rm Dir})_{ij} \sim \langle\phi_u\rangle \lambda^{|H(L_i)+H(N_j)|}\,, \tag{A.9}$$

$$(M_{\nu_s}^{\rm Maj})_{ij} \sim M \lambda^{|H(N_i)+H(N_j)|}\,. \tag{A.10}$$

In $\det M^{(n_s)}$, each $H(N_i)$ appears twice in the exponent, each time with either a plus or a minus sign. Consequently, we obtain the following type of dependence on lepton charges:

$$\det M^{(n_s)} \sim M^{n_s} \lambda^{a_i H(N_i)}\,, \qquad a_i = 0, \pm 2\,. \tag{A.11}$$

Similarly, in $\det M^{(n_s+1)}$, each $H(N_i)$ appears twice in the exponent, each time with a plus or a minus sign. As concerns $H(L_i)$, there are two possibilities: either a single $H(L_i)$ appears twice, each time with either a plus or a minus sign, or two different charges appear once:

$$\begin{aligned}\det M^{(n_s+1)} \sim M^{n_s-1} \langle \phi_u \rangle^2 \; \max \left\{ \lambda^{a_i H(N_i) + bH(L_j)}, \lambda^{a_i H(N_i) + c[H(L_j) + dH(L_k)]} \right\} \\ a_i, b = 0, \pm 2\,, \qquad c, d = \pm 1\,.\end{aligned} \tag{A.12}$$

In a similar manner, we obtain

$$\det M^{(n_s+2)} \sim M^{n_s-2} \langle \phi_u \rangle^4 \lambda^{a_i H(N_i) + b_j H(L_j)}\,, \qquad a_i, b_j = 0, \pm 2\,. \tag{A.13}$$

It is straightforward to see that each of $\det M^{(n_s)}$, $\det M^{(n_s+2)}$ and $[\det M^{(n_s+1)}]^2$ depends on λ^{2n} where n is an integer.

We remind the reader that eq. (A.8) holds only when there is no pseudo-Dirac light neutrino. In other words, it holds in effective $(2, n_s)$ models. Then, $H(L_1)$ does not appear in eqs. (A.11)–(A.13); in particular, the $H(L_i)$-dependence of the second factor in (A.12) is of the form $c[H(L_2) + dH(L_3)]$. Eq. (A.1) implies that this combination of charges is even. Generically, however, this fact has no special consequences and we find

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \sim \frac{m_2^2}{m_3^2} \sim \lambda^{4n} \qquad [(2, n_s) \text{ models}]\,. \tag{A.14}$$

The situation is more constrained if the charges of only two of the sterile neutrinos (say, N_2 and N_3) affect s_{23} and m_2/m_3. This is the $(2, 2)$ model. There are two additional special features in this case:

(i) For $n_s = 2$, we have $\det M^{(n_s=2)} \sim M^2 \lambda^{2|H(N_2)+H(N_3)|}$ which, in particular, allows only $a_i = \pm 2$ in (A.11).

(ii) For any (n, n) model, we have $\det M_\nu = [\det M_\nu^{\rm Dir}]^2$. Consequently, $a_i, b_j = \pm 2$ (and cannot vanish) in eq. (A.13).

As a result of all these features, we now find

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \sim \frac{m_2^2}{m_3^2} \sim \lambda^{8n} \qquad [(2, 2) \text{ models}]\,. \tag{A.15}$$

References

[1] C.D. Froggatt and H.B. Nielsen, *Nucl. Phys.* **B 147** (1979) 277.

[2] M. Leurer, Y. Nir and N. Seiberg, *Nucl. Phys.* **B 398** (1993) 319 [hep-ph/9212278].

[3] M. Dine, A. Kagan and R. Leigh, *Phys. Rev.* **D 48** (1993) 4269 [hep-ph/9304299].

[4] Y. Nir and N. Seiberg, *Phys. Lett.* **B 309** (1993) 337 [hep-ph/9304307].

[5] Y. Grossman, Y. Nir and R. Rattazzi, in *Heavy flavours II*, A.J. Buras and M. Lindner eds., Advanced Series on Directions in High Energy Physics, World Scientific, Singapore 1998, hep-ph/9701231.

[6] M. Leurer, Y. Nir and N. Seiberg, *Nucl. Phys.* **B 420** (1994) 468 [hep-ph/9310320].

[7] Y. Grossman and Y. Nir, *Nucl. Phys.* **B 448** (1995) 30 [hep-ph/9502418].

[8] Y. Fukuda et al., the Super-Kamiokande Collaboration, *Phys. Rev. Lett.* **81** (1998) 1562 [hep-ex/9807003].

[9] J.N. Bahcall, P.I. Krastev and A. Yu. Smirnov, *Phys. Rev.* **D 58** (1998) 096016 [hep-ph/9807216].

[10] P. Binetruy, S. Lavignac and P. Ramond, *Nucl. Phys.* **B 477** (1996) 353 [hep-ph/9802334].

[11] J. Ellis, G.K. Leontaris, S. Lola and D.V. Nanopoulos, *Neutrino textures in the light of Super-Kamiokande data and a realistic string model*, hep-ph/9808251.

[12] Y. Grossman, Y. Nir and Y. Shadmi, *J. High Energy Phys.* **10** (1998) 007 [hep-ph/9808355].

[13] P. Binetruy, S. Lavignac, S. Petcov and P. Ramond, *Nucl. Phys.* **B 496** (1997) 3 [hep-ph/9610481].

[14] M. Tanimoto, hep-ph/9807517; *Phys. Lett.* **B 456** (1999) 220 [hep-ph/9901210].

[15] P. Binetruy and P. Ramond, *Phys. Lett.* **B 350** (1995) 49 [hep-ph/9412385].

[16] Y. Nir, *Phys. Lett.* **B 354** (1995) 107 [hep-ph/9504312].

[17] M. Green and J. Schwarz, *Phys. Lett.* **B 149** (1984) 117.

[18] M. Dine, N. Seiberg and E. Witten, *Nucl. Phys.* **B 289** (1987) 589.

[19] M. Dine, I. Ichinoise and N. Seiberg, *Nucl. Phys.* **B 293** (1987) 253.

[20] H. Dreiner, G.K. Leontaris, S. Lola, G.G. Rocss and C. Scheich, *Nucl. Phys.* **B 436** (1995) 461 [hep-ph/9409369].

JHEP05(1999)023

[21] M.E. Gomez, G.K. Leontaris, S. Lola and J.D. Vergados, *Phys. Rev.* **D 59** (1999) 116009 [hep-ph/9810291].

[22] R. Barbieri, L.J. Hall, D. Smith, A. Strumia and N. Weiner, *J. High Energy Phys.* **12** (1998) 017 [hep-ph/9807235].

[23] L.E. Ibanez and G.G. Ross, *Phys. Lett.* **B 332** (1994) 100 [hep-ph/9403338].

[24] Y. Nir and Y. Shadmi, work in progress.

[25] P.H. Chankowski and Z. Pluciennik, *Phys. Lett.* **B 316** (1993) 312 [hep-ph/9306333].

[26] N. Haba, N. Okamura and M. Sugiura, *The renormalization group analysis of the large lepton flavor mixing and the neutrino mass*, hep-ph/9810471.

[27] G.K. Leontaris, S. Lola, C. Scheich and J. Vergados, *Phys. Rev.* **D 53** (1996) 6381 [hep-ph/9509351].

[28] B.C. Allanach, *Atmospheric neutrino oscillations and maximal $\nu_\mu - \nu_\tau$ mixing in unified models*, hep-ph/9806294.

[29] G.K. Leontaris, S. Lola and G.G. Ross, *Nucl. Phys.* **B 454** (1995) 25 [hep-ph/9505402].

[30] G. Altarelli and F. Feruglio, *J. High Energy Phys.* **11** (1998) 021, [hep-ph/9809596]; *Phys. Lett.* **B 451** (1999) 388 [hep-ph/9812475].